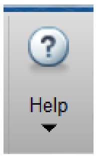
Explore some Fundamental Symbolic Computing syntax:

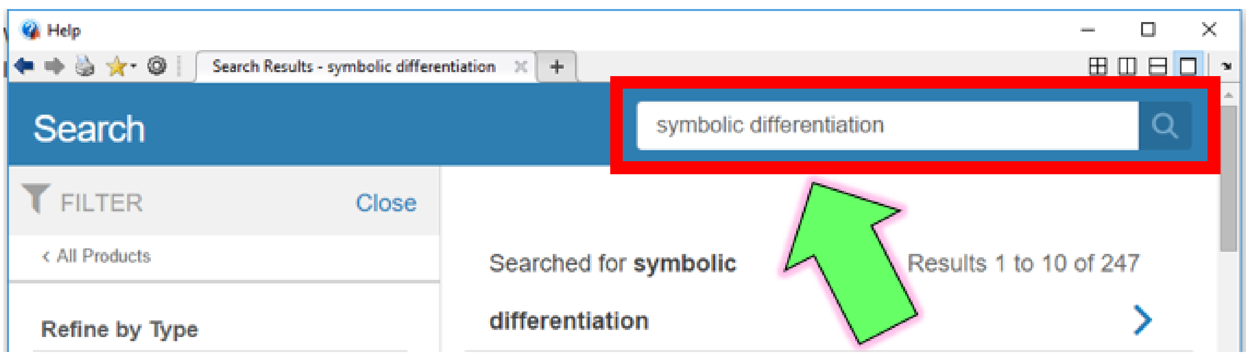
We'll look at some fundamental syntax (patterns) for doing **`Symbolic`** computing. We'll look at how to create and manipulate symbolic expressions. Specifically we, we'll:

- create symbolic expressions
- create symbolic functions
- Plot symbolic expressions
- Differentiate
- Integrate
- computer Laplace transforms
- Solve ODEs
- ******YES:*** apply Lagranges equation to a spring mass damper system

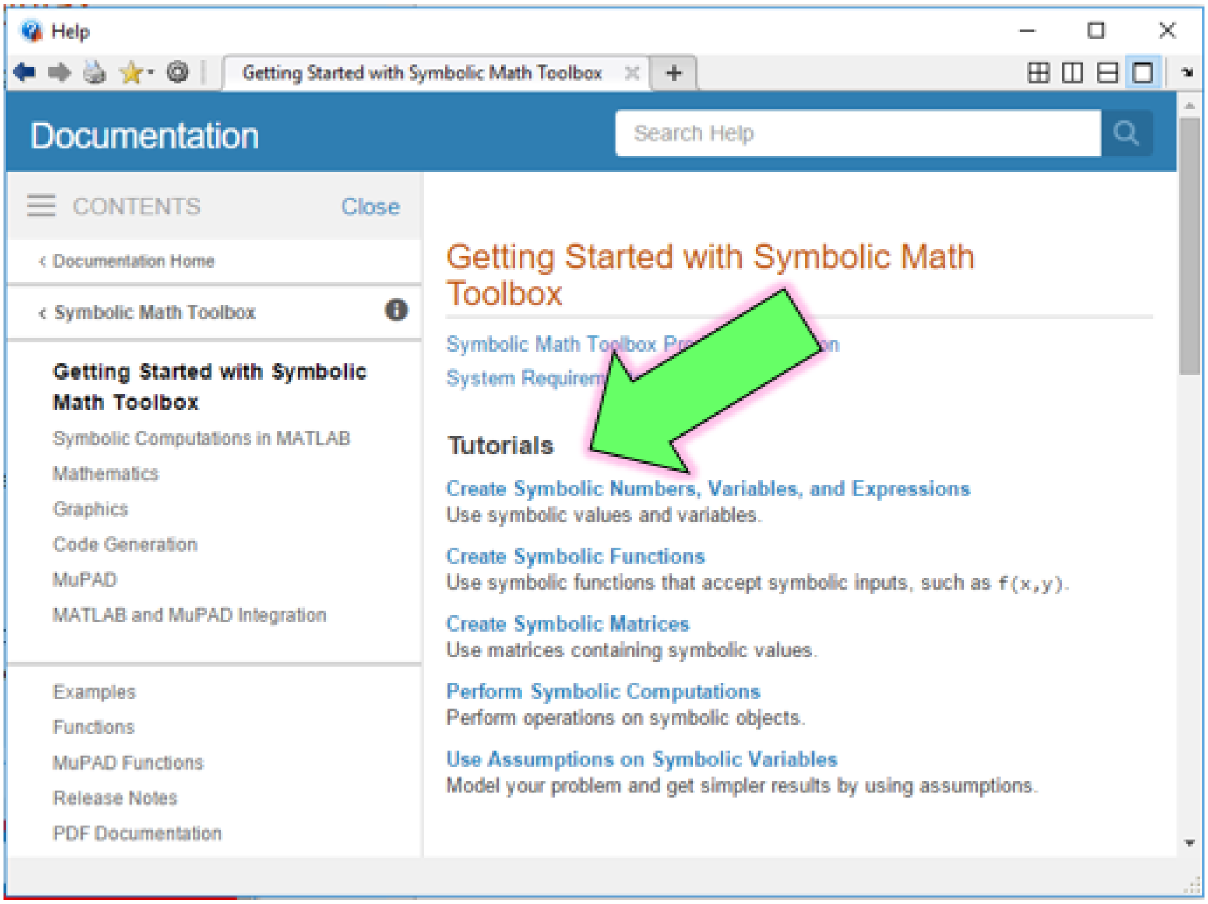
While you're doing this tutorial, don't forget to leverage the MATLAB help browser:

Look for this icon 

on the MATLAB desktop and click on it often ! The help browser lets you search using "plain english" keywords. For example, if you wanted to know how to differentiate symbolic expressions, you could start by typing in this search:



The Help Browser also has a collection of "Getting started" Tutorials !



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A symbolic expression: (aka a sym)

```
syms x y theta
```

And now you can create a symbolic expressions using that **`sym`** variable:

```
f    = exp(-y)/x + 3*x^2
```

```
f =
```

$$\frac{e^{-y}}{x} + 3\,x^2$$

```
R    = [   cos(theta),  -sin(theta),       f;
           sin(theta),   cos(theta),       0;
                0,          0,             1;
      ]
```

```
R =
```

$$\begin{pmatrix} \cos(\theta) & -\sin(\theta) & \frac{e^{-y}}{x} + 3\,x^2 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Using familiar MATLAB indexing syntax, we can extract the 1st row and 3rd column of the R matrix:

```
some_expression = R(1,3)
```

some_expression =

$$\frac{e^{-y}}{x} + 3\,x^2$$

If I have a symbolic expression and I want to evaluate it with some numerical inputs, you would use the **`subs()`** function. Eg: consider the expression stored in g :

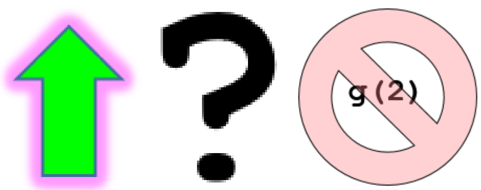
```
g    = 5*x + 7
```

g = $5\,x + 7$

If I want to evaluate this expression with *x=2*, then we'd use the **`subs()`** function:

```
some_value = subs(g,x,2)
```

some_value = 17



A symbolic function: (aka a `symfun`)

Now create some sym variables

```
syms x y
```

We can construct a **`symfun`** symbolic function variable using this syntax:

```
f(x,y) = 5*x + (x^2)*(y^2)
```

f(x, y) = $x^2\,y^2 + 5\,x$

So evaluate:

- f(x=2, y=5): and this should give us: `2*5 + (4)*(25) == 110`

```
Sf_val = f(2,5)
```

Sf_val = 110

If I wanted to "extract" the formulae (or expression) used to define the SYMFUN, here's how:

```
some_expression = formula( f )
```

some_expression = $x^2 y^2 + 5x$

And because we have an expression we can use the **`subs()`** function:

```
subs(some_expression, {x,y}, {2,5})
```

ans = 110

Plotting 2D functions : $y = f(x)$

A function or expression of the form *f(x)*, can be plot/visualized using the **`fplot()`** function:

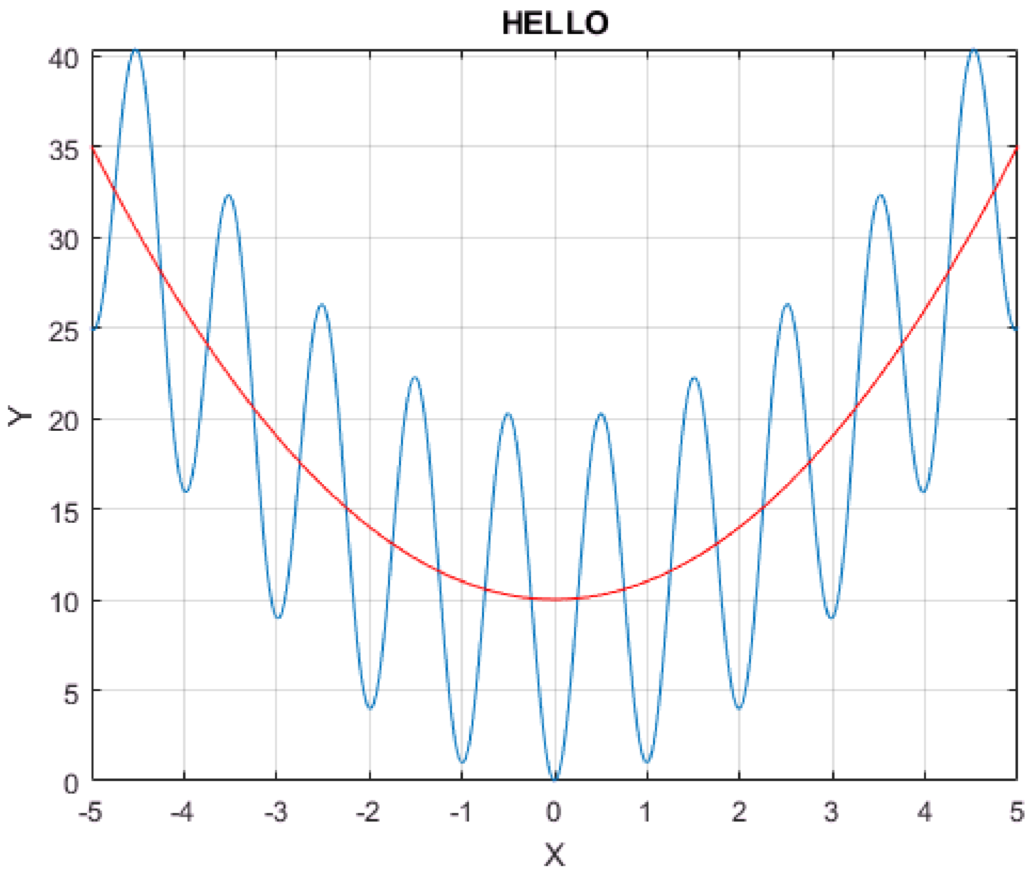
```
syms X
y = 20 + X^2 - 10*(cos(2*pi*X) + 1 );
```

Now plot it using the **`fplot()`** function ... and put add some annotations:

```
figure;
        fplot(y, [-5 5]);

        title('HELLO')
        xlabel('X'); ylabel('Y'); grid('on')
```

and add another function to the plot

```
syms a
z(a) = a^2 + 10;

hold on
fplot(z, [-5 5], '-r')
```



Plotting 3D functions : $z = f(x, y)$

To plot a surface, use the **`fsurf()`** function:

```
syms x y

f = 3*(1-x)^2*exp(-(x^2)-(y+1)^2)...
    - 10*(x/5 - x^3 - y^5)*exp(-x^2-y^2)...
    - 1/3*exp(-(x+1)^2 - y^2);
```

Now plot it:

```
figure;
        h = fsurf(f,[-3 3]);
```

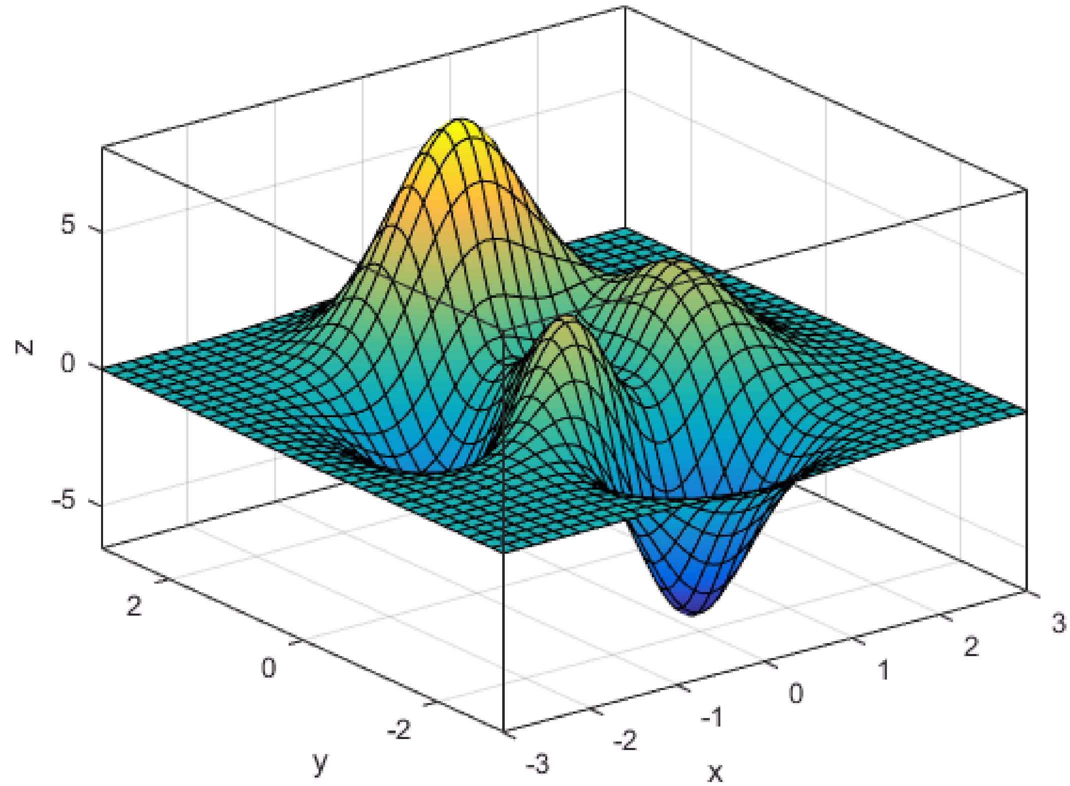
Annotate the plot

```
xlabel('x'); ylabel('y'); zlabel('z');
title_latex = ['$' latex(f) '$'];
title(title_latex,'Interpreter','latex','FontSize', 14)
```

Put a frame around the plot

```
a = gca;
a.Box = 'on';
a.BoxStyle = 'full';
```

$$3\,\mathrm{e}^{-(y+1)^2-x^2}\,(x-1)^2-\frac{\mathrm{e}^{-(x+1)^2-y^2}}{3}+\mathrm{e}^{-x^2-y^2}\,\left(10\,x^3-2\,x+10\,y^5\right)$$



Finding Analytical solutions:

a.) Integration: Let's do this: $f(t) = \int e^{-3t} sin(5t)dt$

```
syms t
dfa_dt = exp(-3*t)*sin(5*t); % the integrand

fa     = int( dfa_dt, t)     % <------ and the INDEFINITE integral is
```

fa =

$$-\frac{\mathrm{e}^{-3\,t}\,\left(5\cos\left(5\,t\right)+3\sin\left(5\,t\right)\right)}{34}$$

b.) Differentiation: Let's do this: $f(t) = cos(2t)e^{5t} \longrightarrow$ so what is $\dfrac{df}{dt}$?

```
fb     = cos(2*t)*exp(5*t);

dfb_dt = diff(fb, t)        % <------ and the derivative is
```

dfb_dt = $5\cos\left(2\,t\right)\,\mathrm{e}^{5\,t}-2\sin\left(2\,t\right)\,\mathrm{e}^{5\,t}$

c.) Laplace transforms: Let's do this: $f(t) = cos(2t)e^{5t} \longrightarrow$ so what is $F(s) = \mathcal{L}\{f(t)\}$?

```
syms s
my_ft = cos(2*t)*exp(5*t);

my_Fs = laplace(my_ft, t, s)
```

my_Fs =

$$\frac{s-5}{(s-5)^2+4}$$

And let's do the inverse too:

```
fc = ilaplace(my_Fs, s, t)
```

fc = $\cos(2\,t)\,e^{5\,t}$

ODEs: solve this $\ddot{x}(t) + 4.\dot{x}(t) + 100.x(t) = 200.u(t-5)$ with

- $x(0) = 5$
- $\dot{x}(0) = 0$

```
% define some SYMBOLIC variables
syms t x(t)

% construct the SYMBOLIC 2nd order ODE
f  = 200*heaviside(t-5);
EQ = 1*diff(x,2) + 4*diff(x,1) + 100*x == f
```

EQ(t) =

$$\frac{\partial^2}{\partial t^2}\,x(t) + 4\,\frac{\partial}{\partial t}\,x(t) + 100\,x(t) = 200\,\mathrm{heaviside}(t-5)$$

Now find the analytical solution:

```
% now find the analytical solution applying our IC's
dx(t) = diff( x(t), t);

x     = dsolve( EQ,   x(0)==5,    ...
                     dx(0)==0,    ...
                    'IgnoreAnalyticConstraints',false);

x     = simplify(x)
```

x =

$$2\,\mathrm{heaviside}(t-5) + 5\,e^{-2\,t}\cos\left(4\,\sqrt{6}\,t\right) + \frac{5\,\sqrt{6}\,e^{-2\,t}\sin\left(4\,\sqrt{6}\,t\right)}{12} - 2\,\mathrm{heaviside}(t-5)\,e^{-2\,t}\,e^{10}$$

where

$$\sigma_1 = 4\,\sqrt{6}\,t - 20\,\sqrt{6}$$

```
% compute the velocity (because we can)
```

```
v     = diff(x,t)
```

v =

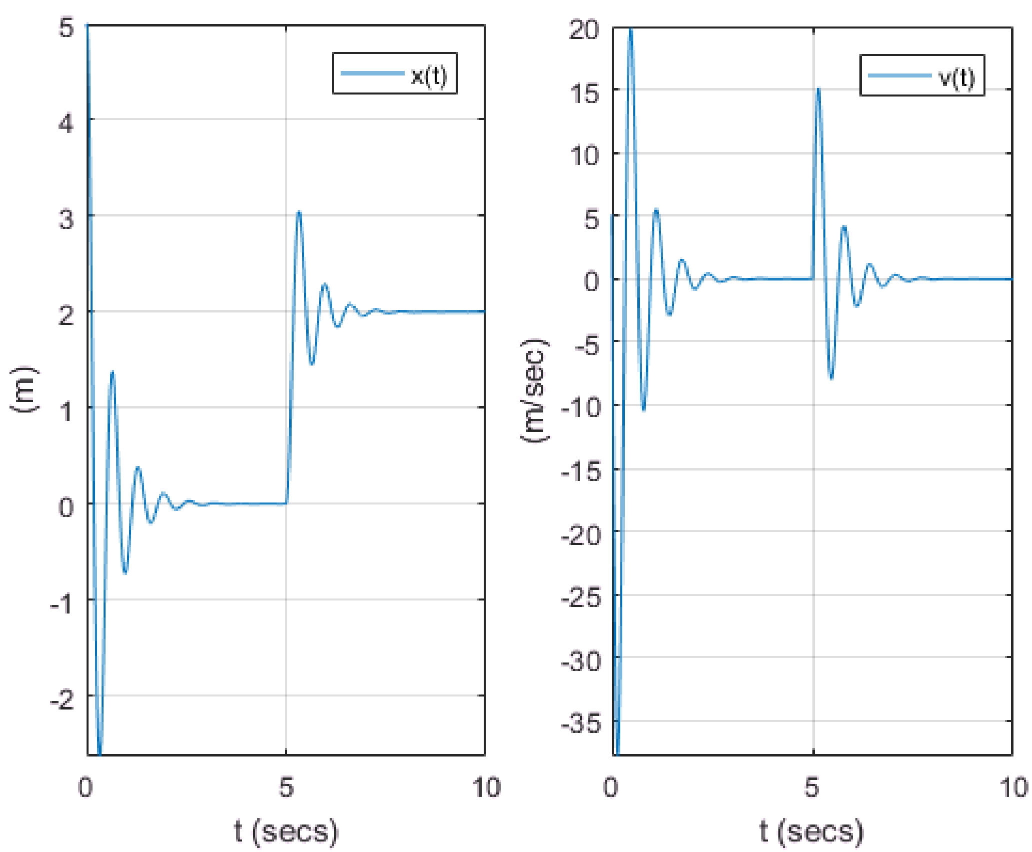
$$2\,\delta(t-5) - \frac{125\,\sqrt{6}\,e^{-2t}\sin\left(4\,\sqrt{6}\,t\right)}{6} - 2\,\delta(t-5)\,e^{-2t}\,e^{10}\cos\left(\sigma_1\right) - \frac{\sqrt{6}\,\delta(t-5)\,e^{-2t}\,e^{10}\sin\left(c\right.}{6}$$

where

$$\sigma_1 = 4\,\sqrt{6}\,t - 20\,\sqrt{6}$$

Plot the solution

```
figure;

subplot(1,2,1)
                fplot( x, [-0.01, 10]);
                       xlabel('t (secs)');  ylabel('(m)');
                       grid('on');          legend('x(t)')
subplot(1,2,2)
               fplot( v, [-0.01, 10]);
                      xlabel('t (secs)');   ylabel('(m/sec)');
                      grid('on');           legend('v(t)')
```



FYI: a symbolic expression can be converted into LaTex code using the **`latex()`** *function. You could then copy and paste this into your own latex files OR into a latex viewer(eg:* ***QuickLaTex.com****) in your web browser(remember to include the $ $)*

```
latex(x)
```

```
ans = 2\, \mathop{\mathrm{heaviside}}\nolimits\!\left(t - 5\right) + 5\, \mathrm{e}^{- 2\, t}\, \cos\!\l
```

Converting symbolic expressions into MATLAB functions:

Say we have a symbolic expression:

```
syms x y

z = x + y;
```

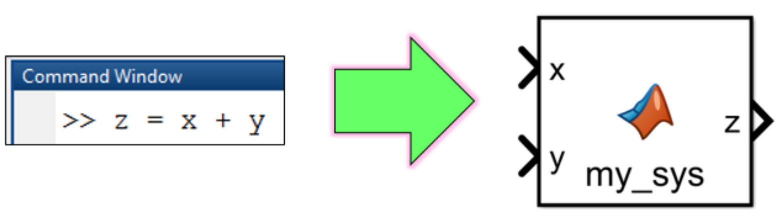
We can convert this symbolic expression into a reusable MATLAB function using the function **`matlabFunction`**():


```
matlabFunction( z,                ...
                'File','my_func', ...
                'Vars', {'x', 'y'});

% look at the autogenerated file
dbtype('my_func.m')
```

```
function z = my_func(x,y)
%MY_FUNC
%    Z = MY_FUNC(X,Y)

%    This function was generated by the Symbolic Math Toolbox version 7.1.
%    16-Feb-2017 09:51:13

z = x+y;
```

Converting symbolic expressions into SIMULINK blocks:

We can convert also convert symbolic expression into Simulink blocks using the function **`matlabFunctionBlock`**():



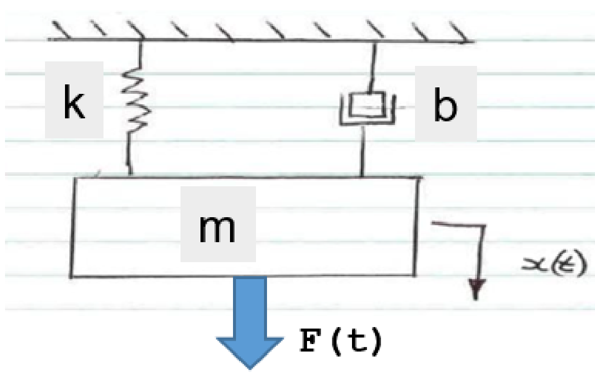

```
FILENAME        = 'SIM_bh_autogen_z';
BLOCK_NAME      = [FILENAME,'/my_THING'];
new_system(FILENAME)
```

```
open_system(FILENAME)

matlabFunctionBlock(BLOCK_NAME, z, 'Vars', {'x', 'y'} );
```

```
Evaluating callback 'PostLoadFcn' for simulink
Callback: setsysloc_simulink(bdroot)
Evaluating callback 'LoadFcn' for simulink/Sources/Waveform Generator
Callback: set_param(gcb,'LoadFlag','1');


Evaluating callback 'LoadFcn' for simulink/Sources/Signal Builder
Callback: sigbuilder_block('load');
Evaluating callback 'LoadFcn' for simulink/Sinks/XY Graph
Callback: sfunxy([],[],[],'LoadBlock')
Evaluating callback 'LoadFcn' for simulink/Model-Wide Utilities/Model Info
Callback: slcm LoadBlock;
Evaluating callback 'LoadFcn' for simulink/Math Operations/Slider Gain
Callback: slideg Load
```

A Case Study - part 1: Deriving the equations of motion

Look at a simple application of Lagrange's equation ... say for simple spring, mass mechanical system: $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = Q$




```
clear; clc
```

Define some Symbolic variables that we can play with:

```
syms t x(t) m k b F
syms THE_X THE_XD THE_XDD
actual_list = [       x,   diff(x,t),   diff(x,t,2)];
HOLDER_list = [   THE_X,   THE_XD,      THE_XDD];
```

Define our system Lagrangian and Generalised force:

```
v  = diff(x,t);  % velocity
KE = 0.5*m*v^2;  % KINETIC energy
PE = 0.5*k*x^2;  % POTENTIAL energy
Q  = F -b*v;     % total NON conservative forces for deltaX
L  = KE - PE     % our Lagrangian
```

```
L(t) =
```

$$\frac{m\left(\frac{\partial}{\partial t}x(t)\right)^2}{2} - \frac{k\,x(t)^2}{2}$$

Now let's start applying Lagranges equation $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x}$:

```
                          % OLD_LIST        NEW_LIST
L_new   = subs(L, actual_list,   HOLDER_list)
```

L_new(t) =

$$\frac{\mathrm{THE}_{\mathrm{XD}}^2\, m}{2} - \frac{\mathrm{THE}_X^2\, k}{2}$$

Our 1st piece is: $\frac{\partial L}{\partial x}$

```
dLdx    = diff(L_new, THE_X)
```

dLdx(t) = $-\mathrm{THE}_X\, k$

Our 2nd piece is: $\frac{\partial L}{\partial \dot{x}}$

```
dLdxdot =  diff(L_new, THE_XD)
```

dLdxdot(t) = $\mathrm{THE}_{\mathrm{XD}}\, m$

Our 3rd piece is: $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}}$

```
                                    % OLD_LIST      NEW_LIST
dLdxdot       =  subs(dLdxdot, HOLDER_list,  actual_list )
```

dLdxdot(t) =

$$m\,\frac{\partial}{\partial t}\,x(t)$$

```
dt_of_dLdxdot = diff(dLdxdot, t)
```

dt_of_dLdxdot(t) =

$$m\,\frac{\partial^2}{\partial t^2}\,x(t)$$

Now put it all together: $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = Q$

```
our_EOM_LHS = dt_of_dLdxdot - dLdx
```

our_EOM_LHS(t) =

$$m \frac{\partial^2}{\partial t^2} x(t) + \mathrm{THE}_X\, k$$

```
our_EOM_RHS = Q
```

our_EOM_RHS(t) =

$$F - b \frac{\partial}{\partial t} x(t)$$

```
our_EOM      = (our_EOM_LHS == our_EOM_RHS)
```

our_EOM(t) =

$$m \frac{\partial^2}{\partial t^2} x(t) + \mathrm{THE}_X\, k = F - b \frac{\partial}{\partial t} x(t)$$

```
                                             % OLD_LIST       NEW_LIST
our_EOM      =  subs(our_EOM, actual_list,  HOLDER_list)
```

our_EOM(t) = $\mathrm{THE}_X\, k + \mathrm{THE}_{\mathrm{XDD}}\, m = F - \mathrm{THE}_{\mathrm{XD}}\, b$

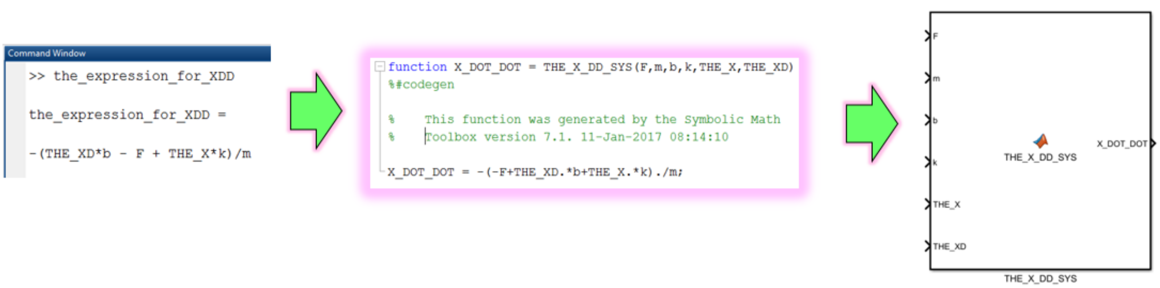
Now solve for $\ddot{x}$:

```
the_expression_for_XDD = solve(our_EOM, THE_XDD)
```

the_expression_for_XDD =

$$-\frac{\mathrm{THE}_{\mathrm{XD}}\, b - F + \mathrm{THE}_X\, k}{m}$$

And now create a Simulink block for this expression:




```
MODEL_NAME         = 'SIM_SMD_WILL_BE_DELETED';
close_system(MODEL_NAME,0)
new_system(MODEL_NAME)
open_system(MODEL_NAME)

matlabFunctionBlock( [MODEL_NAME,'/THE_X_DD_SYS'], the_expression_for_XDD, ...
                         'Vars',     {F, m,b,k,THE_X,THE_XD}, ...
```

```
                    'Outputs',  {'X_DOT_DOT'}   );
```

A Case Study - part 2: Simulating the model

Let's use the model that we just derived, and implement it in Simulink - where we'll numerically solve it. The parameters that we'll use for this Numerical simulation are:

$$\ddot{x}(t) + 4.\dot{x}(t) + 100\,.x(t) = 200.u(t - 5)$$

with

- $x(0) = 5$
- $\dot{x}(0) = 0$

Have a look at our Simulink model .... and NOTE how we use the integrator blocks to integrate: $\ddot{x} \rightarrow \frac{1}{s} \rightarrow \dot{x} \rightarrow \frac{1}{s} \rightarrow x$

```
open_system('bh_a_spring_mass_model')
```